DISTRIBUTION OF SUSPENDED PARTICLES IN A FLUIDIZED BED

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 3, pp. 115–120, 1968

It is important to know the distribution of the number n of particles per unit volume in discussing fluidized-bed processes in chemical plant. The distribution has often been examined qualitatively, but relatively few papers contain quantitative results.

The variation in n with height in the bed approaches uniformity at low flow speeds; n decreases exponentially with height at speeds approaching the limit for loss of particles.

Here a kinetic model [1, 2] for a fluidized bed is used to examine n as a function of height for a gas flow. The theoretical conclusions are in good general agreement with the experimental evidence.

1. Formulation of the problem. We consider the assembly of solid particles and the gas flow as two interacting continuous media. The particles are considered as resembling a gas.

The complete equations of hydrodynamics have been derived [2] for such a system; they differ from similar equations derived elsewhere [3, 4] in having a more complicated structure for the stress tensor for the pseudogas and in requiring the use of an equation for the change in the rms speed of the random particle motion (the pseudotemperature); the latter was previously [3, 4] neglected. See [2] concerning some other features of this model.

Consider a bed of solid particles of diameter σ, which has an initial depth h and is laterally unbounded. A real bed can be represented in this schematic way only if the lateral size L ≫ h, when the effects of the walls can be neglected. We also assume that convective movement is absent and that the mean macroscopic velocity can be taken as zero. Some comments are made below on the stability of this state.

The axes of the fixed Cartesian coordinate system are such that the x_1-axis is perpendicular to the plane of the distribution grid at the base and is directed vertically upward, while the x_2- and x_3-axes lie in the plane of that grid. Then the dimensionless variables

$$x_1 = hx, \quad \theta = \rho_d v_0 Q^2 T, \quad N = nv_*, \quad \varkappa = \frac{v_0}{v_*} \tag{1.1}$$

give us the following system of hydrodynamic equations:

$$K\xi \frac{d}{dx}\left[\frac{NT}{1-N} P_2(N)\right] + \xi \frac{d}{dx}\left[\frac{1-N}{1-{}^{11}/_{16}N}(1-\varkappa N)^{-3.75}\frac{T_0}{\sqrt{T}}\right] =$$

$$= N\frac{1-\gamma(1-\varkappa N)^{4.75}}{(1-\varkappa N)^{4.75}},$$

$$T_0 = DL(R_0)\varkappa^2 N^2 (1-N)^2 [P_3(N)]^{-1}(1-\varkappa N)^{-4},$$

$$\frac{3}{16}K\xi^2 \frac{d}{dx}\left[\frac{\Lambda(N)}{1-N}\sqrt{T}\frac{dT}{dx}\right] = N(1-\varkappa N)^{-3.75}(T-T_0), \tag{1.2}$$

in which

$$\xi = \frac{5\sigma}{12h\sqrt{\pi}}, \quad K = \frac{Q\sigma\rho_d}{18\nu_0\rho_0 H(R_0)}, \quad \Phi_0 = \frac{18\nu_0\rho_0}{\sigma^2\rho_d},$$

$$\gamma = \frac{g}{Q\Phi_0 H(R_0)}, \quad R_0 = \frac{Q\sigma}{\nu_0},$$

$$L(R_0) = \left[4.75 - \frac{R_0 H'(R_0)}{H(R_0)}\right], \tag{1.3}$$

where v_* is the volume per particle in close packing in space, v_0 is the volume of a particle, Q is the volume of gas flowing through unit cross section of the grid in unit time, ρ_0 is the gas density, ρ_d is particle density, ν_0 is the kinematic viscosity of the gas, and g is the acceleration due to gravity.

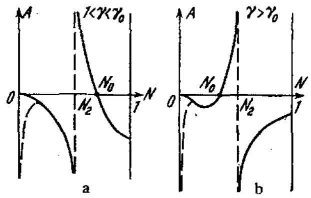

Fig. 1

The forms of Λ(N), $P_2(N)$, and $P_3(N)$ have been given [2]. We use a semiempirical relation [5] for the force of interaction between the components, which gives $H(R_0)$ as

$$H(R_0) = 1 + 10^{-2}\left(R_0 + \sqrt{R_0^2 + 200R_0}\right).$$

The basis for this relation has been discussed [6, 7].

The problem is to find a solution to (1.2) that satisfies the conditions for conservation of the number of particles in the bed:

$$\int_0^\infty N(x)\,dx = 1. \tag{1.4}$$

We must also specify conditions for the pseudotemperature T at the boundary with the grid. A special discussion is needed to establish the form of these conditions, and the exact form of the conditions is not relevant to this paper.

2. Bed with large initial h. If $\sigma \approx 10^{-2}$ cm and $h \approx 10^2$ cm, while $\nu_0 \approx 10^{-2}$ poise, $\rho_0/\rho_d \approx 10^{-3}$, and $Q \approx 10^2$ cm/sec, we have $K \approx 10^4$, $\xi \approx 10^{-4}$, $K\xi \approx 1$, $K\xi^2 \approx 10^{-4}$. We neglect terms of order ξ and $K\xi^2$ to get

$$T = T_0(N), \tag{2.1}$$

$$\frac{dx}{dN} = K_\xi DL(R_0)\varkappa^2 A(N)[1-\gamma(1-\varkappa N)]^{-4.75},$$

in which A(N) is a function whose explicit form is easily established.

Figure 1a, b shows the behavior of the right-hand side of (2.1) for various γ. The point N_0 is deduced from the condition $A(N_0) = 0$.

Equation (2.1) resembles (1.2) in having two exact solutions:

$$N_1 \equiv 0, \quad N_2 \equiv \varkappa^{-1}(1 - \gamma^{-0.21}), \tag{2.2}$$

but one of these does not satisfy (1.4). Equation (2.1) will give an ad-

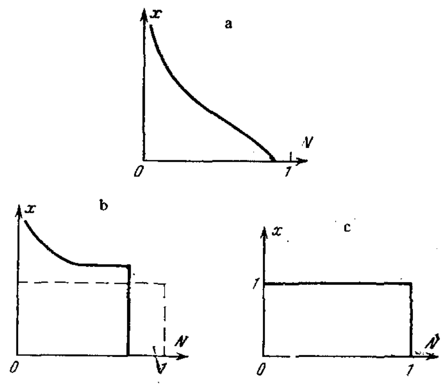

Fig. 2

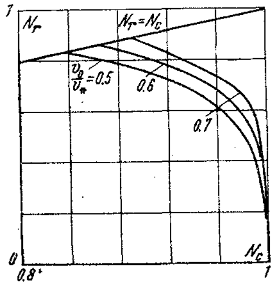

Fig. 3

equate approximation only for $N \gg \sqrt{\xi}$; the terms on the left in the first equation in (1.2) are of like order for smaller N. If the appropriate correction is included, the behavior of A(N) near N = 0 alters as shown by the dashed line in Fig. 1a, b. It can be shown that the resulting singularity in the behavior is summable and that

$$\int_0^N N \frac{dx}{dN} dN < \infty \qquad (N > 0).$$

Consider the form of the N(x) defined by (2.1) subject to (1.4).

The behavior of A(N) for $\gamma < 1$ indicates that there is no solution that satisfies (1.4), which means physically that the fluidized bed cannot exist at such high Q, since these are such as to carry off even single particles.

Also, if

$$1 < \gamma < \gamma_0 = (1 - \varkappa N_0)^{-4.75}, \tag{2.3}$$

there is a continuous solution to (2.1) that satisfies (1.4), in which case Fig. 2a shows N(x).

If, on the other hand,

$$\gamma_0 < \gamma < \gamma_{\max} = (1 - \varkappa)^{-4.75}, \tag{2.4}$$

there is no continuous solution to (2.1) that satisfies (1.4). To derive the appropriate discontinuous solution we make the following change of variable in (1.2):

$$x' = x\xi^{-1/2}. \tag{2.5}$$

We now discard terms of order ξ to get

$$NTP_2(N) = (1 - N)\, p_0, \tag{2.6}$$

$$\frac{3}{16} K \frac{d}{dx'}\left[\frac{\Lambda(N)}{1-N} T^{1/2} \frac{dT}{dx'}\right] = N(1 - \varkappa N)^{-3.75}(T - T_0),$$

in which p_0 is a constant that must be determined during the solution of the problem.

If $x' \to \pm\infty$, the system will take up a definite condition and dT/dx' approaches zero.* Let N_c and T_c be the values of N(x') and T(x') for $x' \to -\infty$ and let N_r and T_r be the same for $x' \to \infty$. It can be shown from (2.6) that there exists a pair of values $N_c > N_r$ such as to satisfy

$$p_0 = \frac{N_c T_0(N_c) P_2(N_c)}{1 - N_c} = \frac{N_r T_0(N_r) P_2(N_r)}{1 - N_r}, \tag{2.7}$$

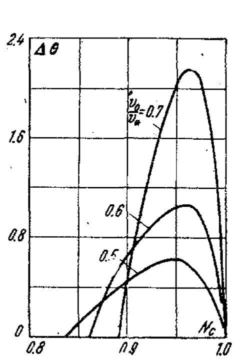

Fig. 4

where

$$T_c = T_0(N_c), \quad T_r = T_0(N_r), \tag{2.8}$$

and N_c may be chosen arbitrarily in the range $[N_0, 1]$. We note also that $N_r \to 0$ if $N_c \to 1$. We return to (2.1) to find that

$$N_c = \varkappa^{-1}(1 - \gamma^{-0.21}), \tag{2.9}$$

and for p_0 we get

$$p_0 = \frac{1 - \gamma^{-0.21}}{\varkappa - (1 - \gamma^{-0.21})} \times \tag{2.10}$$

$$\times\, T_0[\varkappa^{-1}(1 - \gamma^{-0.21})]\, P_2[\varkappa^{-1}(1 - \gamma^{-0.21})].$$

The function N(x) then takes the form shown in Fig. 2b. For $\gamma = \gamma_{\max}$, which corresponds to the minimum possible Q, N(x) takes the form of a step function (Fig. 2c).

This solution means that, for $\gamma > \gamma_0$, there is a sharp boundary between the homogeneous part of the fluidized bed and the very dilute part. This boundary vanishes as Q increases and the bed becomes everywhere inhomogeneous. This picture is in good qualitative agreement with the observed one apart from the region directly adjoining the grid [8], where the behavior of the particles is determined by the interaction with the grid and requires a special discussion outside the scope of this paper.

Figures 3–6 give results calculated from

$$\Delta\theta = [T_0(N_r) - T_0(N_c)]\,\Gamma^{-1}, \quad \theta = T_0(N)\,\Gamma^{-1}, \tag{2.11}$$

$$P_\gamma = p_0 v_* \Gamma^{-1}\; \Gamma = {}^1/_3\, Dx^2 L(R_0).$$

Constant D in these expressions has to be deduced by experiment. $T_0(N)$ has been derived in another way [9]. The numerical result are different because different equations of state were used for the pseudogas, namely Eyring's equations in [9] and van der Waals' equation here.

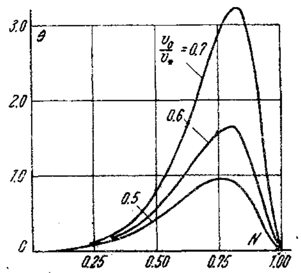

Fig. 5

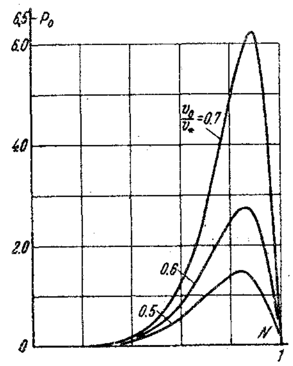

Fig. 6

*This problem resembles that in deriving the relations at a density discontinuity via the Navier-Stokes equations for a viscous compressible gas.

Figure 7 illustrates clearly the region of existence of the fluidized bed and the region of existence of a homogeneous fluidized bed. We introduce

$$R_{\min}=\frac{(1-\varkappa)^{4.75}N_{Ar}}{18+0.6\sqrt{(1-\varkappa)^{4.5}N_{Ar}}},\qquad N_{Ar}=\frac{g\sigma^2}{18\nu_0^2}\left(\frac{\rho_d}{\rho_0}\right),$$

$$R_*=\frac{\gamma_0^{-1}N_{Ar}}{18+0.6\sqrt{\gamma_0^{-1}N_{Ar}}},$$

$$R_{\max}=\frac{N_{Ar}}{18+0.6\sqrt{N_{Ar}}}. \tag{2.12}$$

The Archimedes number N_{Ar} will be fixed for given properties of the bed and gas flow. We draw in Fig. 7 a straight line N_{Ar} parallel to the R-axis to get the region of existence of the fluidized bed and the critical value R that divides this region into two parts: $R_{\min} < R < R_*$, where a state of uniform density exists, and $R_* < R < R_{\max}$, where the bed is inhomogeneous throughout its thickness.

The following are some properties of the upper bound to the existence of a homogeneous state. The mean kinetic energy of a particle alters across this boundary $x = x_0$ (Fig. 8), and the following amount of work must be performed to take a particle from the region $x < x_0$ to $x > x_0$:

$$\delta A = T_r - T_c. \tag{2.13}$$

3. Distribution for Q large. The previous section shows that N will be inhomogeneous throughout the bed if Q is large enough. Also, the maximum N will not exceed N_2 for $1 < \gamma < \gamma_0$, which $N_2 \to 0$ for $\gamma \to 1$. We put

$$T = T_0(N)E \tag{3.1}$$

and expand the functions of N in (1.2) as series, retaining only terms of the least order of smallness in N. We get

$$\omega\xi\frac{d}{dx}(NE^{-1/2})=(1-\gamma)N,\qquad \omega=\varkappa\,[DL(R_0)]^{1/2},$$
$$\frac{3}{16}K\xi^2\omega\Lambda(0)\frac{d}{dx}\left(N^3E^{1/2}\frac{dE}{dx}+2N^2E^{3/2}\frac{dN}{dx}\right)=(E-1)N^3. \tag{3.2}$$

We seek a solution to (3.2) in the form

$$E=\text{const},\quad N=Ae^{-Ax}. \tag{3.3}$$

We substitute (3.3) into (3.2) to get equations for E and A. We eliminate A to get

$$\frac{9K\Lambda(0)}{8\omega}(\gamma-1)^2E^{5/2}=(E-1). \tag{3.4}$$

Also, γ is close to unity for Q large, since Q is close to $Q_{\max}$ and

$$E=1+\frac{9K\Lambda(0)}{8\omega(Q_{\max})}(\gamma-1)^2+O(|\gamma-1|^3). \tag{3.5}$$

Similarly,

$$A=\frac{\gamma-1}{\xi\omega(Q_{\max})}+O(|\gamma-1|^3). \tag{3.6}$$

We therefore finally get

$$N=Ae^{-Ax},\quad T=\frac{\gamma-1}{\xi}\omega Ae^{-2Ax},\quad P_{11}=A\xi\omega e^{-Ax}. \tag{3.7}$$

The calculations can be compared with experiment [10]. Figure 9 shows A as a function of Q for beds composed of identical particles but having different initial thicknesses, where the points are from [10], while the solid lines are theoretical relations derived as follows. We expand $\gamma - 1$ as a power series in $(Q_{\max} - Q)/Q_{\max}$ and stop at terms of the first order of smallness to get

$$A=\frac{12\sqrt{\pi}}{5}\left(\frac{h}{\sigma}\right)\frac{\varkappa}{\sqrt{D}}\left[\frac{5.75-L(R_{\max})}{L(R_{\max})}\right]\frac{Q_{\max}-Q}{Q_{\max}}. \tag{3.8}$$

Figure 9 shows that A is very nearly linearly dependent on Q, and also [10] that A is very much dependent on the initial depth (the upper line in Fig. 9 corresponds to the smaller thickness), which arises for the following reasons. The gas pressure difference across the bed increases with the initial thickness. The gas density has a nonlinear relation to the pressure, so the mean gas density in the bed is dependent on the initial thickness, and hence $Q_{\max}$ is affected. We can estimate the density change as follows.

If we neglect the weight of the gas within the fluidized bed, the following [2] is the equation for conservation of momentum in projection on a direction perpendicular to the grid:

$$\frac{d}{dx_1}[\Pi+p+P_{11}]=\rho_d\varkappa Ng, \tag{3.9}$$

in which Π is the pressure in the gas flow, p is the pressure of the pseudogas, and P_{11} is the normal stress in the pseudogas. We integrate (3.9) with limits zero and infinity, using the dependence of p and P_{11} on N, to get by virtue of (1.4) that

$$\Delta\Pi=\Pi(\rho)|_{x_1=0}-\Pi(\rho)|_{x_1=\infty}=\rho_d\varkappa gh. \tag{3.10}$$

The pressure difference across the bed is independent of Q within the range in which the fluidized bed exists [11]. The ρ of (3.10) is the local gas density.

Then the mean gas density in the bed is

$$\Pi'(\rho_0)(\rho|_{x=0}-\rho|_{x=\infty})=\rho_d\,\varkappa gh. \tag{3.11}$$

We assume that $\Pi = a\rho^\lambda$. As $\Pi(0)$ and $\Pi(\infty)$ are known, and the latter is small relative to $\rho_d\varkappa gh$, we finally get from (3.11) that

$$\rho_0\approx c(\rho_d\,\varkappa gh)^{1/\lambda}. \tag{3.12}$$

Constant c is dependent on the properties of the gas.

This relationship gives a correct general description of the dependence of A on the initial thickness.

4. Stability of the homogeneous state. The results of sections 2 and 3 show that the particle distribution for Q small is very different

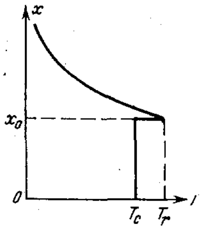

Fig. 8

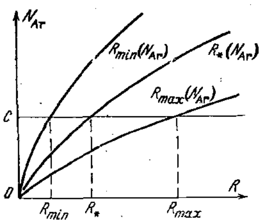

Fig. 7

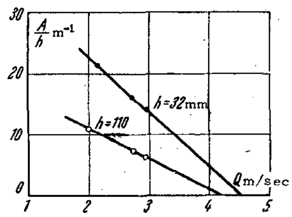

Fig. 9

from that for Q large. Technical processes are usually conducted at low Q, so we consider only conditions such that there is a region of homogeneity in accordance with the conclusions of section 2.

For Q small we have

$$\frac{p_0}{\rho_d g h} \ll 1. \tag{4.1}$$

Then it can be shown that the stability study of [2] for a simple model is entirely analogous to that for a homogeneous layer of considerable thickness when the complete equations of hydrodynamics [2] are used. The homogeneous state is unstable, and the bed will necessarily give rise to convective motion of the particles. This instability has been observed repeatedly [11].

The characteristic forms of the convection are very much dependent on the details of the interaction with the walls and grid, as well as on the shape of the apparatus, the mode of distribution of the gas, etc. The convection produces a change in the height distribution of the particles, but the effects are small if Q exceeds the minimum value only slightly.

I am indebted to V. G. Levich for valuable discussions and to E. Markaryan and L. Kholod for assistance with the calculations.

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14 August 1967 Moscow